LOSS MECHANISMS IN DIELECTRIC LOADED RESONATORS

Kawthar A. Zaki and C. Chen

University of Maryland, Department of Electrical Engineering
College Park, MD 20742

ABSTRACT

Analysis is presented of resonators consisting of a section of a dielectric loaded waveguide shorted at both ends. The analysis includes resonant frequency calculation, mode charts and unloaded Q computation. Numerical results are presented for the unloaded Q's of various modes, as a function of the resonator parameters. Effects of losses in different parts of the resonator wall on the unloaded Q are discussed, and methods of improving these Q's are explored.

INTRODUCTION

Many applications require the availability of microwave resonators with low loss and small size. Dielectric loaded waveguide resonators are suitable for such applications as highly temperature stable oscillators [1],[2], low noise microwave synthesizer [3], and bandpass filters [4]-[6]. This paper presents properties of resonators consisting of a section of a dielectric loaded waveguide shorted at both ends with particular emphasis on the (ohmic) loss mechanisms that affect their unloaded Q's. Previous analysis has considered losses only due to radiation from unshielded resonators [7]. Explicit analytical expressions for the unloaded Q's are derived for the hybrid modes, with the contributions of the losses in each section of the resonator's boundary separately identified. Numerical results are presented for different modes.

RESONATOR GEOMETRY, RESONANT FREQUENCY AND MODE CHARTS

The dielectric loaded resonator geometry under consideration is shown in Fig. 1. Analysis of this type of resonators is very useful for the understanding of the more general case where the dielectric rod is shorter than the total cylinder length L.

The structure of Fig. 1 can resonate in various modes which correspond to the modes of propagation in the cylindrical loaded waveguide discussed in Reference [8]. These modes can be axially symmetric (i.e. transverse electric TE_{0mn} or transverse magnetic TM_{0mn}), or hybrid modes HE_{1mn}. The resonant frequency of any of these modes is computed from applying the boundary conditions that the tangential electric fields must vanish on the ends of the resonator. This condition yields

$$\sin \beta L = 0, \quad \beta L = n\pi, \quad n = 1,2,... \qquad (1)$$

where β is the propagation constant of the mode in an infinite waveguide with the same cross section as the resonator ($\beta^2 = -\gamma^2$). Determination of the resonant frequency involves solving the characteristic equation [8] for the wave number ξ_1:

$$G_n(\xi_1 a) = U_n^2 a^2 \gamma^2 + k_0^2 a^2 V_n W_n = 0 \qquad (2)$$

where

$$\xi_1^2 = k_1^2 + \gamma^2; \quad \zeta_2^2 = -(k_2^2 + \gamma^2);$$

$$k_1^2 = \varepsilon_{r_1} k_0^2; \quad k_2^2 = \varepsilon_{r_2} k_0^2; \quad k_0^2 = \omega^2 \mu_0 \varepsilon_0;$$

$$U_n = n\, J_n(\xi_1 a)\left[\frac{1}{\xi_1^2 a^2} + \frac{1}{\zeta_2^2 a^2}\right];$$

$$V_n = \left[\frac{J_n'(\xi_1 a)}{\xi_1 a} + \frac{P_n'(\zeta_2 a)}{\zeta_2 a}\right]; \quad \alpha_n = \frac{-U_n}{V_n};$$

$$W_n = \left[\varepsilon_{r_1} \frac{J_n'(\xi_1 a)}{\xi_1 a} + \varepsilon_{r_2} \frac{R_n'(\zeta_2 a)}{\zeta_2 a}\right]$$

$$P_n(\zeta_2 r) = J_n(\xi_1 a)\left[\frac{K_n(\zeta_2 r) I_n'(\zeta_2 b) - I_n(\zeta_2 r) K_n'(\zeta_2 b)}{K_n(\zeta_2 a) I_n'(\zeta_2 b) - I_n(\zeta_2 a) K_n'(\zeta_2 b)}\right];$$

$$R_n(\zeta_2 r) = J_n(\xi_1 a)\left[\frac{K_n(\zeta_2 r) I_n(\zeta_2 b) - I_n(\zeta_2 r) K_n(\zeta_2 b)}{K_n(\zeta_2 a) I_n(\zeta_2 b) - I_n(\zeta_2 a) K_n(\zeta_2 b)}\right];$$

$J_n(\cdot)$, $I_n(\cdot)$ and $K_n(\cdot)$ are the Bessel functions, modified Bessel functions of first and second kinds, respectively.

Fig. 2 is a mode chart for a typical resonator. This mode chart is similar in shape to the case for an unshielded resonator presented in reference [7].

This material is based upon work supported by the National Science Foundation under Grant No. ECS-8320249.

0149-645X/85/0000-0465 $01.00 © 1985 IEEE

UNLOADED Q CALCULATIONS

Determination of the resonators unloaded Q's involves the calculation of the energy stored U in the resonator, and also the total power lost W_L in the metallic walls as well as in the dielectric. The unloaded Q is then calculated from:

$$Q = \frac{\omega_o U}{W_L} \tag{3}$$

where ω_o is the resonant angular frequency.

Although the calculation of the Q is conceptually simple, the details are rather involved. In the following closed form expressions are given of the relevant quantities.

First the energy stored U is computed as the sum of the stored energies both inside the dielectric ε_{r_1} (U_1) and in the region between the dielectric and the conducting waveguide walls U_2. Thus

$$U = U_1 + U_2 \tag{4}$$

where

$$U_1 = \frac{\pi L \varepsilon_o \varepsilon_{r_1}}{4}[J_n^2(\xi_1 a)\{\frac{a^2}{2}(1-\frac{n^2}{\xi_1^2 a^2})(1+\beta^2\frac{1+\alpha^2}{\xi_1^2}) + \frac{2\beta^2 n\alpha}{\xi_1^4}\} + J_n'^2(\xi_1 a)\{\frac{a^2}{2}(1+\beta^2\frac{1+\alpha^2}{\xi_1^2})\} + J_n(\xi_1 a)J_n'(\xi_1 a)\,\beta^2\frac{1+\alpha^2}{\xi_1^3}] \tag{5}$$

$$U_2 = \frac{\pi L \varepsilon_o \varepsilon_{r_2}}{4}[-P_n^2(\zeta_2 b)\frac{b^2}{2}\frac{\beta^2\alpha^2}{\zeta_2^2}(1+\frac{n^2}{\zeta_2^2 b^2}) + Q_n'^2(\zeta_2 b)\frac{b^2}{2}(\frac{\beta^2}{\zeta_2^2}-1) - J_n^2(\xi_1 a)\{\frac{a^2}{2}(1+\frac{n^2}{\zeta_2^2 a^2})(1-\beta^2\frac{1+\alpha^2}{\zeta_2^2}) + (\frac{2\beta^2 n\alpha}{\zeta_2^4}\} + Q_n'^2(\zeta_2 a)\frac{a^2}{2}\{1-\frac{\beta^2}{\zeta_2^2} - P_n'^2(\zeta_2 a)\frac{a^2\alpha^2}{2} - J_n(\xi_1 a)\frac{\beta^2 a}{\zeta_2^3}\,Q_n'(\zeta_2 a) + \alpha^2 P_n'(\zeta_2 a)\}] \tag{6}$$

The power lost in the resonator walls and in the dielectric are computed based on the low loss assumptions, that the fields are unperturbed by the losses. The current density in the conducting walls are determined from the tangential magnetic fields, and dielectric loss is proportional to the stored energy in the dielectric times the loss tangent. Accordingly, the dielectric loss is merely proportional to U_1 given in Equation (5), while the conductor loss W_c is expressed as:

$$W_c = W_s + 2W_B \tag{7}$$

where W_s is the loss in the side walls (or circumference) of the resonator and W_B is the loss in each of the ends (base and top). W_B is further separated as:

$$W_B = W_{B_1} + W_{B_2} \tag{8}$$

where W_{B_1} is the loss in the base region $0<r<a$ covered by the dielectric (ε_{r_1}) and W_{B_2} is the loss in the annular base region $a<r<b$ under the dielectric (ε_{r_2}). These are evaluated to be:

$$W_s = \frac{\pi b L R_s}{4}[\frac{\alpha^2\beta^2}{\omega^2\mu^2}P_n^2(\zeta_2 b) + \frac{1}{\zeta_2^4}\{\omega\,\varepsilon_2\zeta_2 Q_n'(\zeta_2 b) + \frac{\alpha\beta^2 n}{\omega\mu b}P_n(\zeta_2 b)\}^2] \tag{9}$$

$$W_{B_1} = \frac{\pi R_s}{2\xi_1^4}[\frac{\xi_1 a^2}{2}(1-\frac{n^2}{\xi_1^2 a^2})(\omega^2\varepsilon_1^2 + \frac{\alpha^2\beta^4}{\omega^2\mu^2}) + \frac{2\alpha\beta^2\varepsilon_1 n}{\mu}\}J_n^2(\xi_1 a) + \frac{\xi_1^2 a^2}{2}\{\omega^2\varepsilon_1^2 + \frac{\alpha^2\beta^4}{\omega^2\mu^2}\}J_n'(\xi_1 a) + \xi_1 a\{\omega^2\varepsilon_1^2 + \frac{\alpha^2\beta^4}{\omega^2\mu^2}\}\,J_n(\xi_1 a)J_n'(\xi_1 a)] \tag{10}$$

$$W_{B_2} = \frac{\pi R_s}{2\zeta_2^4}[\frac{-b^2\zeta_2^2}{2}(1+\frac{n^2}{\zeta_2^2 b^2})\frac{\alpha^2\beta^4}{\omega^2\mu^2}P_n^2(\zeta_2 b) + \frac{b^2\zeta_2^2}{2}\omega^2\varepsilon_2^2\,Q_n'^2(\zeta_2 b)$$

$$+ \{\frac{a^2\zeta_2^2}{2} (1+ \frac{n^2}{\zeta_2^2 a^2})(\omega^2\varepsilon_2^2 + \frac{\alpha^2\beta^4}{\omega^2\mu^2}) -$$

$$\frac{2\alpha\beta^2\varepsilon_2 n}{\mu}\} J_n^2 (\xi_1 a)$$

$$- \frac{a^2\zeta_2^2}{2} \{\omega^2\varepsilon_2^2 Q_n'^2 (\zeta_2 a) + \frac{\alpha^2\beta^4}{\omega^2\mu^2} P_n'^2(\zeta_2 a)\}$$

$$-\zeta_2 a\{\omega^2\varepsilon_2^2 Q_n' (\zeta_2 a) + \frac{\alpha^2\beta^4}{\omega^2\mu^2} P_n'(\zeta_2 a)\}J_n(\xi_1 a)] \qquad (11)$$

where R_S is the conductor's surface resistance.

NUMERICAL RESULTS, DISCUSSIONS AND CONCLUSIONS

To ascertain the effects of the losses in the various parts of the resonator, it is convenient to express the total unloaded Q in terms of the factors Q_S, Q_{B1}, Q_{B2} and Q_D corresponding to the losses W_S, W_{B1}, W_{B2}, and W_D respectively:

$$\frac{1}{Q} = \frac{1}{Q_S} + \frac{1}{Q_{B1}} + \frac{1}{Q_{B2}} + \frac{1}{Q_D} \qquad (12)$$

Variations of each of these Q's (normalized by (δ/λ_0), where δ is the skin depth and λ is the wavelength in dielectric at resonance), with 2 a/L) and (b/a) as a parameter are shown in Figs. 3 to 6 for the HE_{111} mode. For (b/a) > 1.4 the loss W_S becomes negligibly small. Fig. 6 shows that Q_D will always be approximately equal to $1/\tan\delta$ (here δ is the loss tangent of ε_{r_1}). Variation of the total normalized unloaded Q (due to conductor loss only) with (2 a/L) is shown in Fig. 7. Comparison between Fig. 5 and Fig. 7 shows that for (2 a/L) > 1, the loss W_{B1} dominates the other conductor losses W_{B2} and W_S. Qualitatively, to minimize the loss W_{B1} the resonator could be constructed so that the conducting wall ends are not contacting the dielectric; thus the fields outside the dielectric would decay in the axial direction greatly reducing the current density (and the loss) on these end walls. This, however, would perturb the modal fields in the resonator; the resonant frequencies and unloaded Q's will not be as simple to compute as in the present case. When all the conducting walls are far enough from the dielectric, the conductor losses will be negligible with respect to the dielectric loss, and therefore the unloaded Q in this case will be approximately equal to $1/\tan\delta$, regardless of the mode.

Figure 8 shows the total normalized Q due to the conductor loss only for the first few hybrid modes.

REFERENCES

(1) H. Abe et al, "A highly stabilized low noise GaAs FET integrated oscillator with a dielectric resonator in the C-band", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-26, pp.156-163, March 1978.

(2) S. Tatsuguchi et al, "An integrated 18 GHz receiver front end using a dielectric resonator stabilized generator" in ICC-79, Conf. Record, Vol. 1, 262.1-262.5.

(3) G. D. Alley and H. C. Wang, "An ultra low-noise microwave synthesizer", Digest of the 1974 IEEE International Microwave Symposium, pp. 147-149.

(4) J. K. Plourde and D. F. Linn, "Microwave dielectric resonator filters using $Ba_2 Ti_2 O_{20}$ Ceramics", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-27, pp. 233-238, March 1979.

(5) A. E. Atia and R. R. Bonetti, "Generalized dielectric resonator filters", COMSAT Technical Review, Vol. 11, No. 2, Fall 1981, pp. 321-343.

(6) S. J. Fiedziusko, "Dual-mode dielectric resonator loaded cavity filters", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-30, pp. 1311-1316, September 1982.

(7) Y. Kobayashi and S. Tanaka, "Resonant modes of a dielectric rod resonator short circuited at both ends by parallel conducting plates" IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-28, No. 10, October 1980, pp. 1077-1085.

(8) K. A. Zaki and A. E. Atia, "Modes in dielectric-loaded waveguides and resonators", IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-31, No. 12, December 1983, pp. 1039-1045.

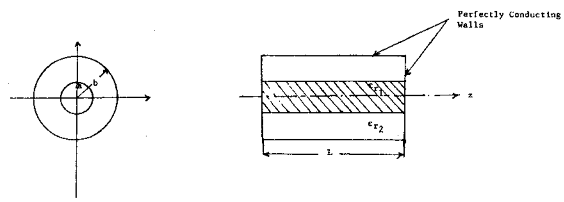

Fig. 1 Dielectric Loaded Resonator consisting of a short circuited section of a dielectric loaded waveguide.

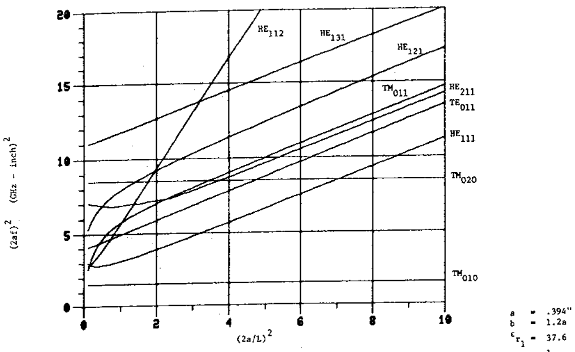

Fig. 2 Mode chart of dielectric loaded Resonator

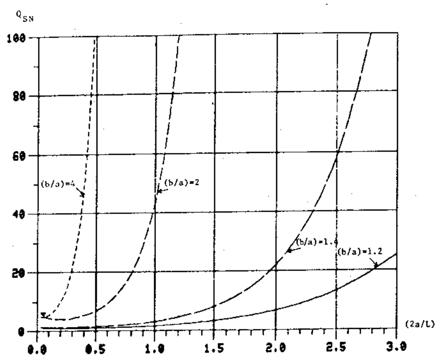

Fig. 3 Q_{SN} for HE_{111} mode vs. (2 a/L)

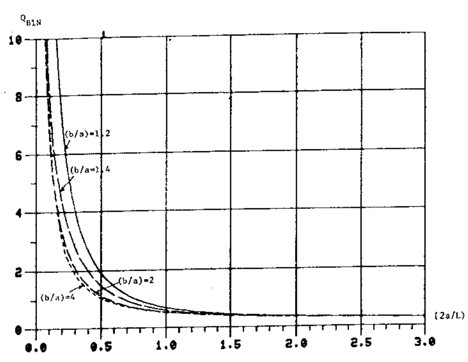

Fig. 4 Q_{B1N} for HE_{111} mode vs. (2 a/L)

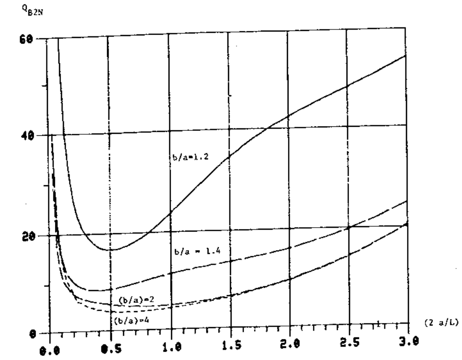

Fig. 5 Q_{B2N} for HE_{111} mode vs. (2 a/L)

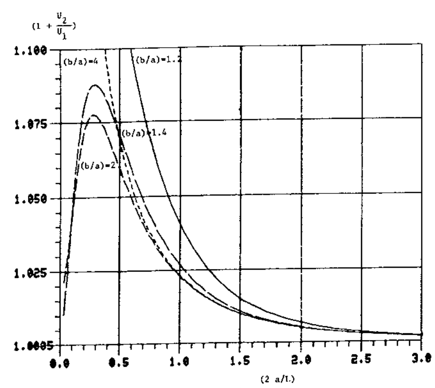

Fig. 6 Variation of Q_D tanδ with (2 a/L) for HE_{111} mode.

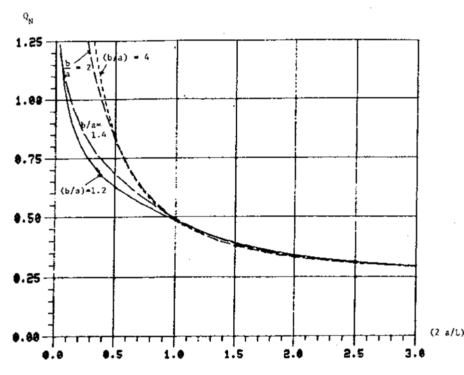

Fig. 7 Unloaded Q_N for HE_{111} mode vs. (2 a/L)

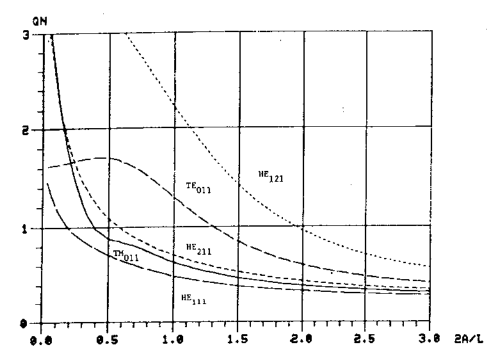

Fig. 8 Q_N for different mode vs. (2 a/L)